DESIGN OF MACHINE ELEMENTS

Module-VI

Nivish George

RSET
RAJAGIRI SCHOOL OF ENGINEERING & TECHNOLOGY

Shafts

Transmission shaft usually refers to a rotating machine element, circular in cross-section, which supports transmission elements like gears, pulleys and sprockets and transmits power

Categories of transmission shafts

- Axle
- Spindle
- Countershaft
- Jackshaft
- Line shaft

Design of shafts on strength basis

Shaft subjected to pure axial tensile load

$$d_0 = \left(\frac{32M}{\pi\sigma_d}\left[\frac{1}{1-K^4}\right]\right)^{\frac{1}{3}}$$

(Eqn. 3.4b) with K=0 for solid shafts

Shaft subjected to pure torsional load

$$d_0 = \left(\frac{16T}{\pi\tau}\left[\frac{1}{1-K^4}\right]\right)^{\frac{1}{3}}$$

(Eqn. 3.4a) with K=0 for solid shafts

Combined torsion and bending (maximum normal stress theory)

$$d = \left(\frac{16}{\pi\sigma_{max}}\left(M+\sqrt{M^2+T^2}\right)\times\left[\frac{1}{1-K^4}\right]\right)^{\frac{1}{3}}$$

(Eqn. 3.5a)

Combined torsion and bending (maximum shear stress theory)

$$d = \left(\frac{16}{\pi\tau}\left(\sqrt{M^2+T^2}\right)\times\left[\frac{1}{1-K^4}\right]\right)^{\frac{1}{3}}$$

(Eqn. 3.5b)

Design of shafts on Rigidity basis

Torsional rigidity: Ability to resist twist under the action of an external torque

Lateral rigidity: Ability to resist deflection under the action of external forces

Rigidity

- Torsional rigidity

$$d = \left(\frac{584TL}{G\theta}\right)^{\frac{1}{4}} \quad \text{(Eqn. 3.2)}$$

$$K = \frac{d_i}{d_o}$$

$$\theta = \left(\frac{584TL}{(d_0^4 - d_i^4)G}\right) \quad \text{(Eqn. 3.2)}$$

- Lateral rigidity

Design of shafts on strength basis (Fluctuating load)

Shaft subjected to pure axial tensile load

$$d_0 = \left(\frac{32C_m M}{\pi\sigma_b}\right)^{\frac{1}{3}}$$

Shaft subjected to pure torsional load

$$d_0 = \left(\frac{16C_t T}{\pi\tau}\right)^{\frac{1}{3}}$$

Combined torsion and bending (maximum normal stress theory)

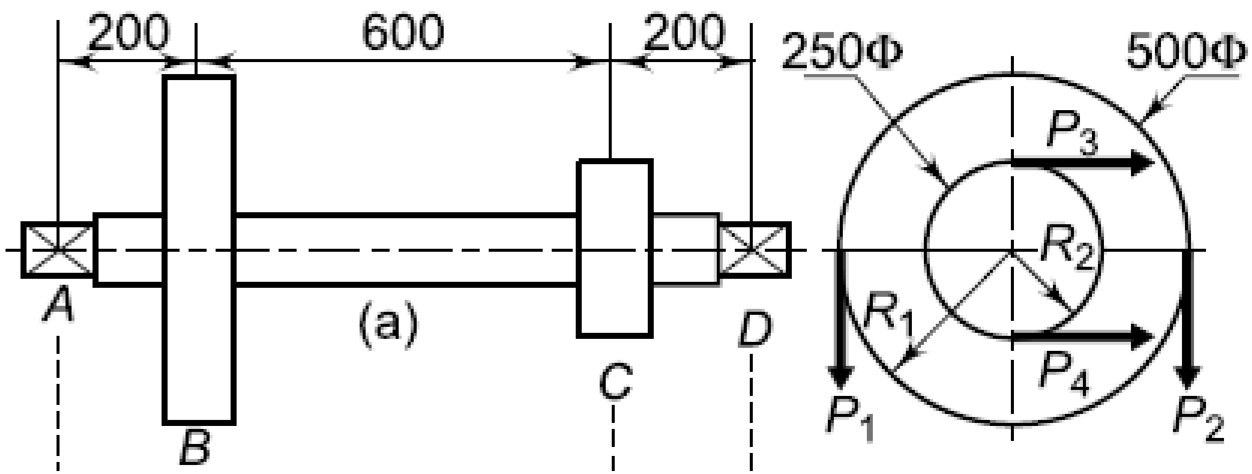
$$d_0 = \left(\frac{16}{\pi\sigma_{max}}\left(\left(C_m M + \sqrt{(C_m M)^2 + (C_t T)^2}\right)\right) \times \left[\frac{1}{1 - K^4}\right]\right)^{\frac{1}{3}}$$ (Eqn. 3.6a)

Combined torsion and bending (maximum shear stress theory)

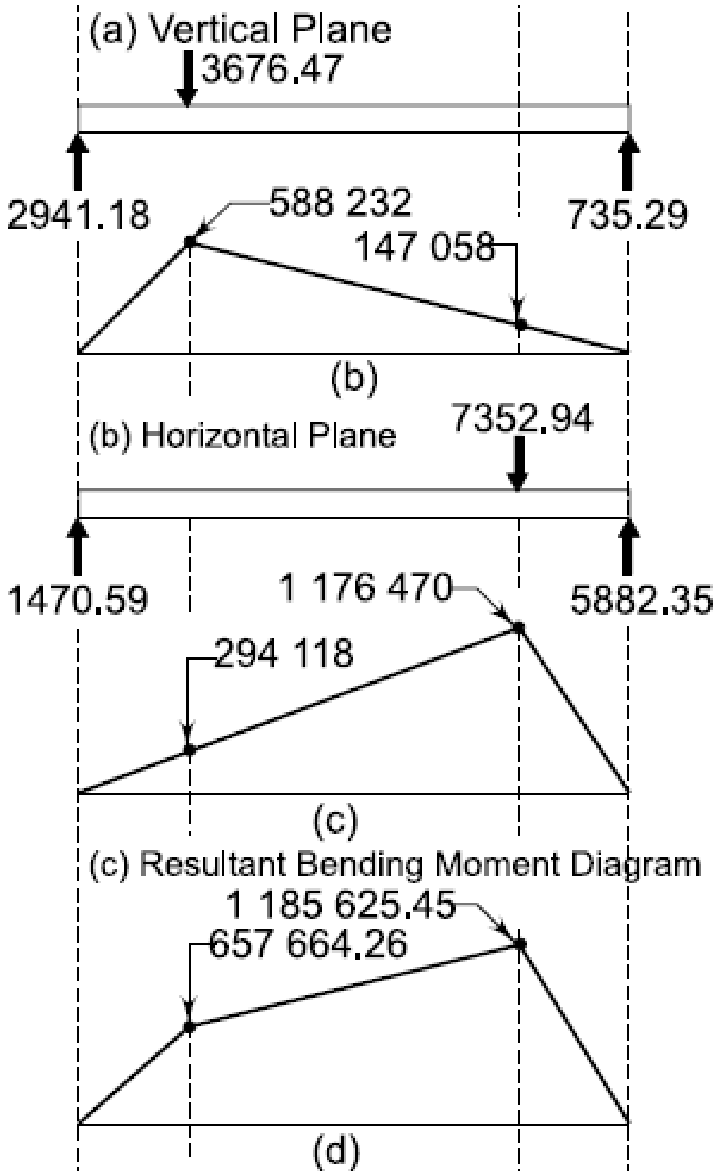
$$d_0 = \left(\frac{16}{\pi\tau}\left(\sqrt{(C_m M)^2 + (C_t T)^2}\right) \times \left[\frac{1}{1 - K^4}\right]\right)^{\frac{1}{3}}$$ (Eqn. 3.6b)

Problem 6.1

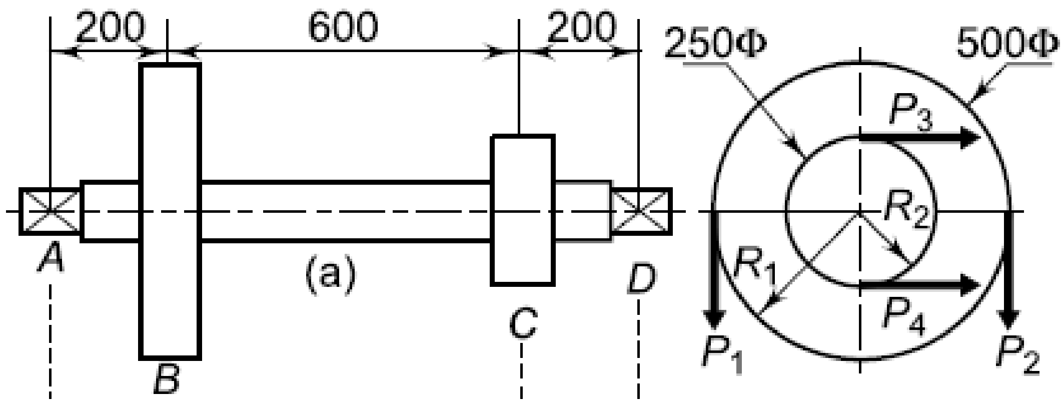
The layout of a transmission shaft carrying two pulleys B and C are supported on bearings A and D is shown in Figure. Power is supplied to the shaft by means of a vertical belt on the pulley B, which is then transmitted to the pulley C carrying a horizontal belt. The maximum tension in the belt on the pulley B is 2.5 kN. The angle of wrap for both pulleys is 180^0 and the coefficient of friction is 0.24. The shaft is made of plain carbon steel 30C8 (S_{yt}=400 N/mm^2) and the factor of safety is 3. Determine the shaft diameter on strength basis.



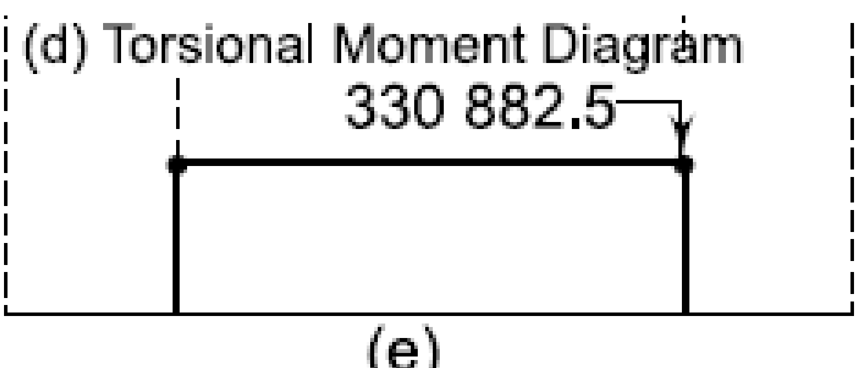
Problem 6.1



(a) Vertical Plane
3676.47
2941.18
735.29
588 232
147 058
(b)
(b) Horizontal Plane
7352.94
1470.59
5882.35
1 176 470
294 118
(c)
(c) Resultant Bending Moment Diagram
1 185 625.45
657 664.26
(d)



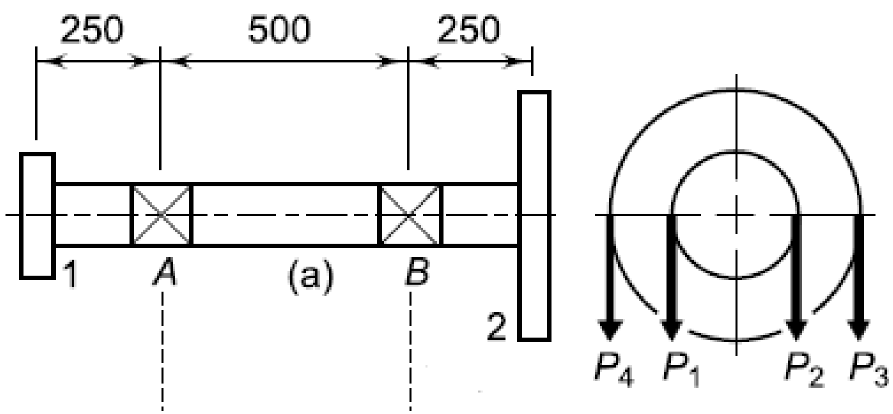
200
600
200
250Φ
500Φ
A
B
C
D
(a)
P_1
P_2
P_3
P_4
R_1
R_2



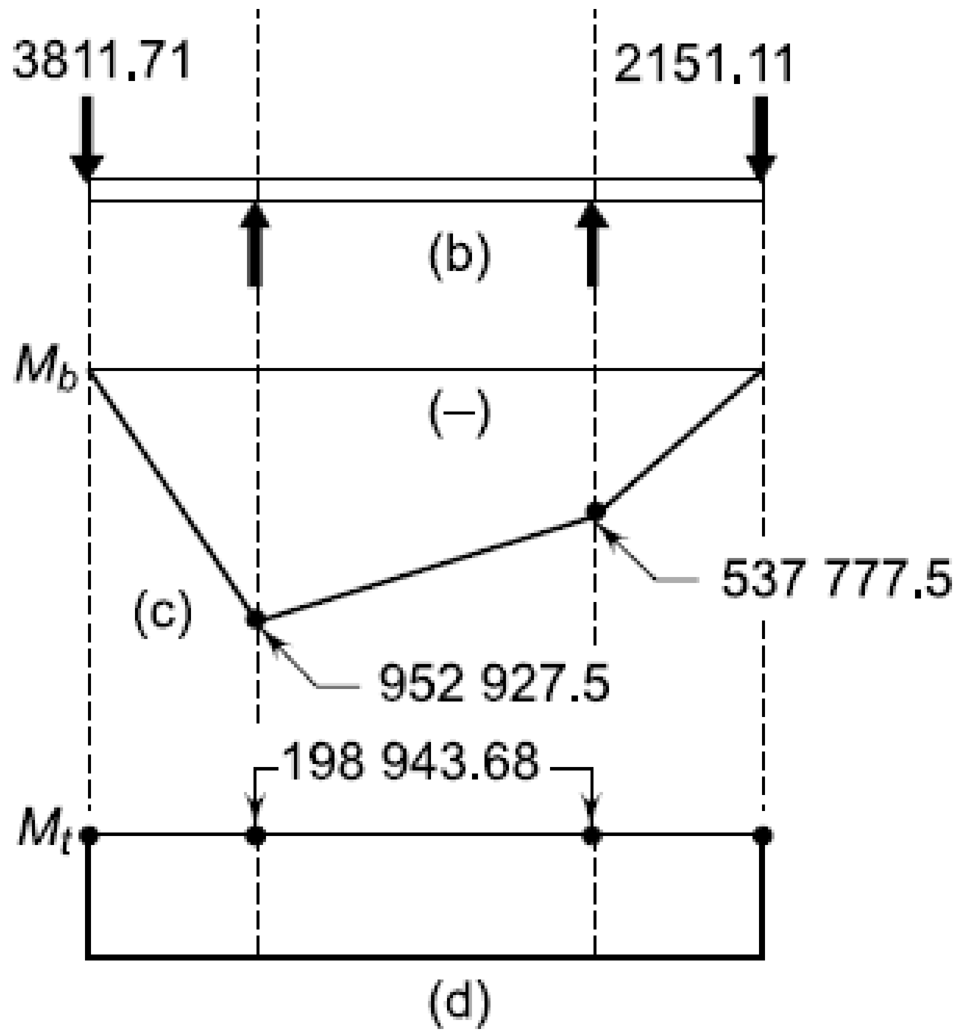
(d) Torsional Moment Diagram
330 882.5
(e)

Problem 6.2

The layout of a shaft carrying two pulleys 1 and 2, and supported on two bearings A and B is shown in Figure. The shaft transmit 7.5kW power at 360 rpm from the pulley 1 to the pulley 2. The diameters of pulleys 1 and 2 are 250 mm and 500 mm, respectively. The masses of the pulleys 1 and 2 are 10 kg and 30 kg, respectively. The belt tensions act vertically downward and the ratio of belt tensions on the tight side to slack side for each pulley is 2.5:1. The shaft is made of plain carbon steel 40C8 (S_{yt} = 380N/mm^2) and the factor of safety is 3. Estimate suitable diameter of shaft. If the permissible angle of twist is 0.5^0 per metre length , calculate the shaft diameter on the basis of torsional rigidity. Assume G=79300 N/mm^2.

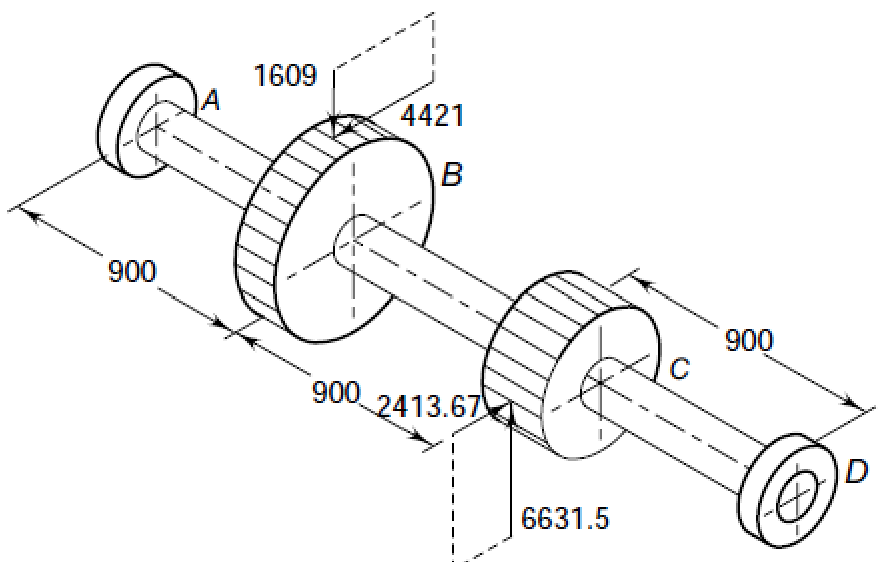


Problem 6.2

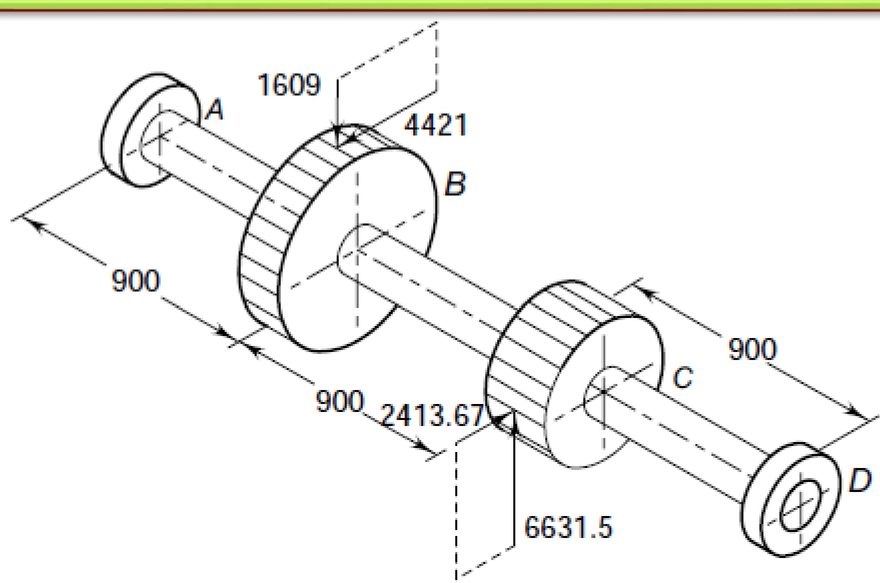


Problem 6.3

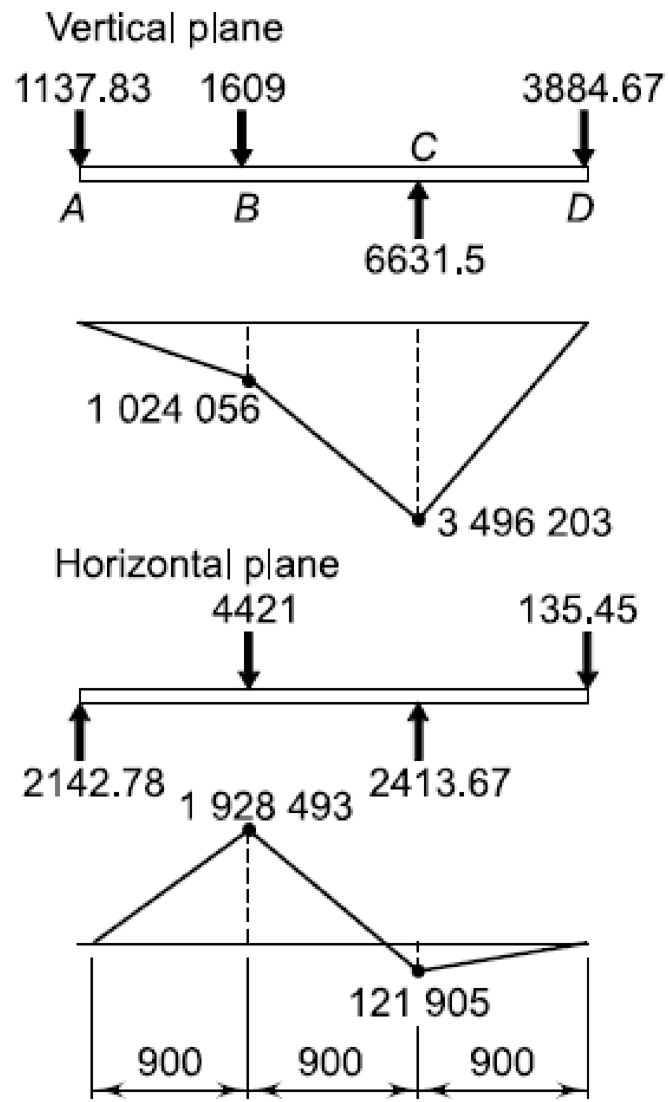
A layout of an intermediate shaft of a gear box supporting two spur gears B and C is shown in Figure. The shaft is mounted on two bearings A and D. The pitch circle diameters of gears B and C are 900 and 600 mm, respectively. The material of the shaft is steel FeE 580 (S_{ut} = 770 and S_{yt} = 580 N/mm^2). The factors C_m and C_t are 1.5 and 2.0, respectively. Determine the shaft diameter using the A.S.M.E code. Assume that the gears are connected to the shaft by means of keys.



Problem 6.3



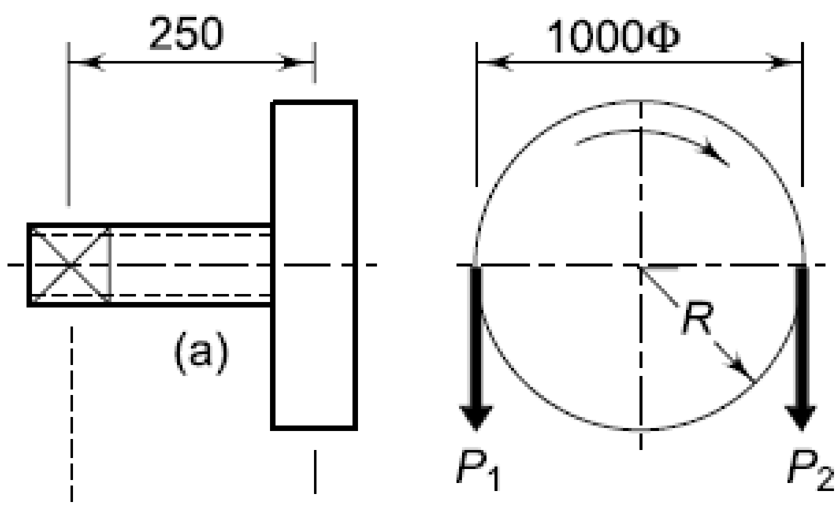
1609
4421
A
B
900
900
2413.67
C
900
D
6631.5



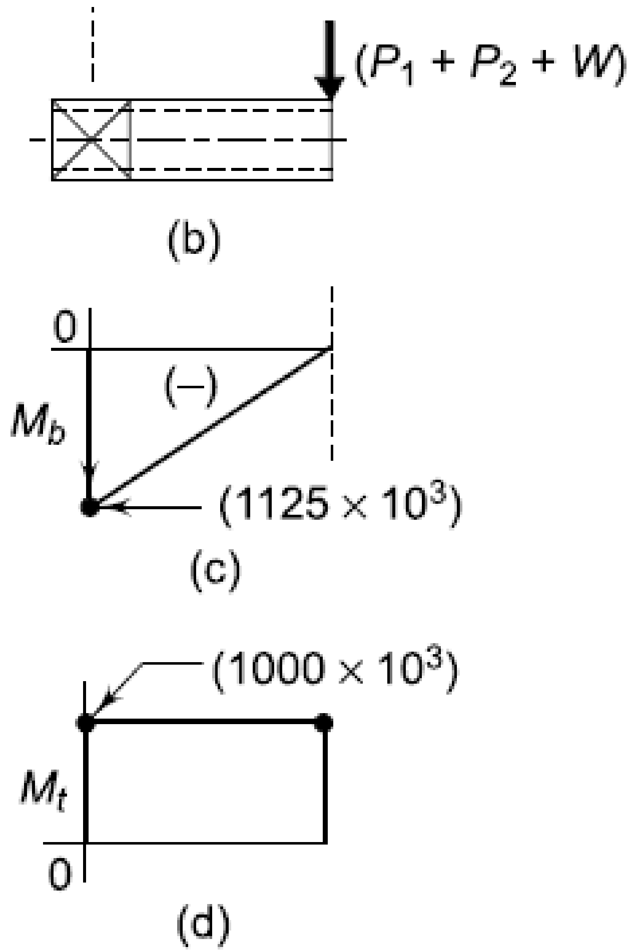
Vertical plane
1137.83
1609
3884.67
C
A
B
D
6631.5
1 024 056
3 496 203
Horizontal plane
4421
135.45
2142.78
1 928 493
2413.67
121 905
900
900
900

Problem 6.4

A hollow transmission shaft, having inside diameter of 0.6 times the outside diameter is made of plain carbon steel 40C8 (S_{yt}=380N/mm^2) and the factor of safety is 3. A belt pulley 1000mm in diameter, is mounted on the shaft which overhangs the left hand bearing by 250 mm. The belts are vertical and transmit power to the machine shaft below the pulley. The tension on the tight side and slack sides of the belt are 3kN and 500 N. The angle of wrap of the belt on the pulley is 180^0. Calculate the outside and inside diameters of the shaft.



Problem 6.3

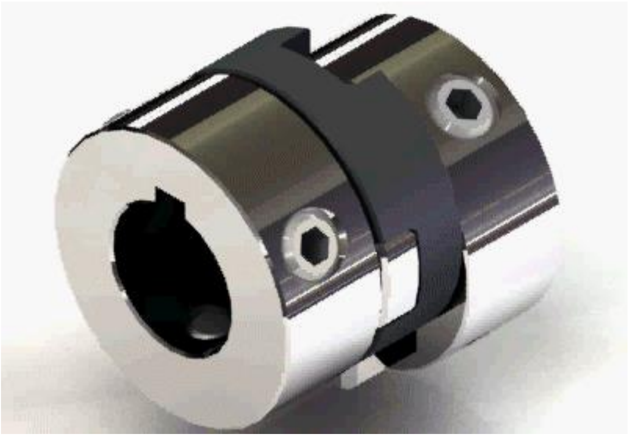


Problem 6.5

A solid shaft of diameter d is used in power transmission. Due to modification of the existing system, it is required to replace the solid shaft by a hollow shaft of the same material and equally strong in torsion. Further, the weight of the hollow shaft per metre length should be half of the solid shaft. Determine the outer diameter of the hollow shaft in terms of d

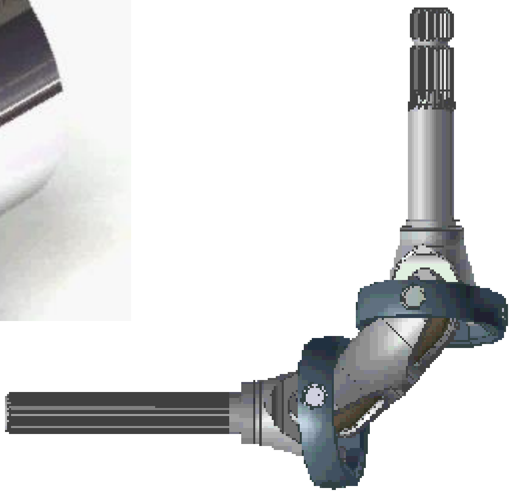
Couplings

A coupling can be defined as a mechanical device that permanently joints two rotating shafts to each other



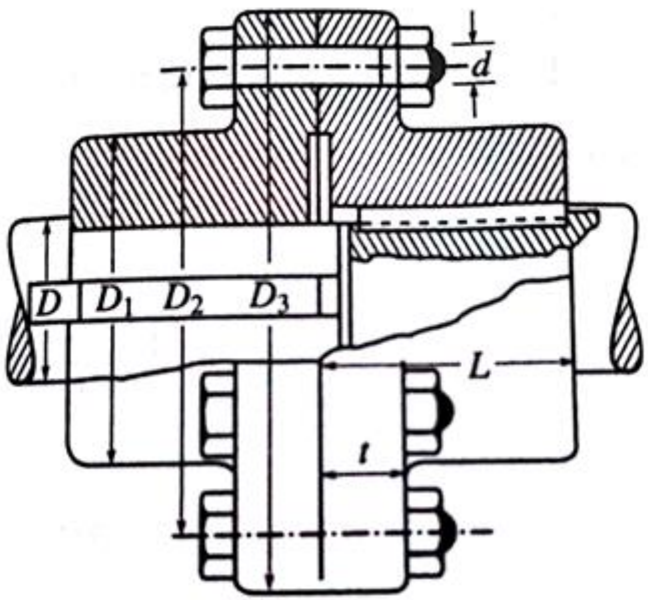
Types

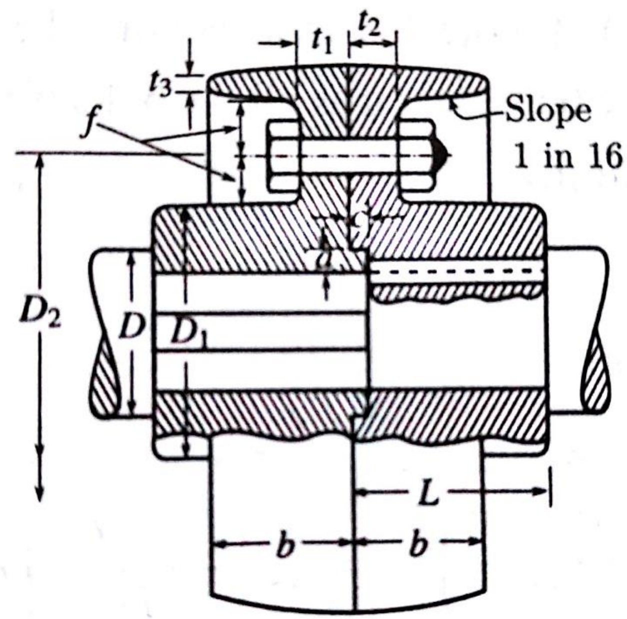
- Oldham coupling
- Hooke's coupling



- Rigid or flexible coupling

Rigid Coupling





$D: Diameter\ of\ the\ shaft$

$L: length\ of\ hub = 1.25\ to\ 1.5D$ $(Eqn\ 13.1g)$

$D_1: Hub\ Diameter = 2D$ $(Eqn\ 13.1d)$

$D_2: Diameter\ of\ bolt\ circle = 3D$ $(Eqn\ 13.1e)$

$D_3: Diameter\ of\ Flange = 4D$ $(Eqn\ 13.1f)$

$t: Flange\ thickness = 0.35D + 9\ mm$ $(Eqn\ 13.1h)$

$t_3: thickness\ of\ protecting\ rim = 0.10D + 5\ mm$ $(Pg. 251)$

$i: No.of\ bolts = \frac{1}{40}D + 2\ \ to\ \ \frac{3}{80}D + 2\ or\ 0.02D + 3$ $(Eqn\ 13.1a\ and\ 13.1b)$

Design Procedure: Rigid Flange Coupling

- Obtain the shaft diameter. Standard shaft sizes can be selected from Table 3.5a, Page 57
- Find the dimensions of flanges by empirical relations on Pg 251 and Pg 252
- The inner and outer diameters of the hub are D and D_1.

$$\tau = \frac{Tr}{J}$$

$$J = \frac{\pi(D_1^4 - D^4)}{32}$$

$$r = \frac{D_1}{2}$$

- Shear force on flange = $\pi D_1 t \times \tau$
- Resistive torque = Shear force $\times \frac{D_1}{2}$

$$T = \frac{1}{2}\pi D_1^2 t\tau \qquad \text{(Eqn. 13.2e)}$$

Design Procedure: Rigid Flange Coupling

- Decide the number of bolts based on empirical relationship

$$i: No.\,of\ bolts = \frac{1}{40}D + 2 \quad to \quad \frac{3}{80}D + 2\ or\ 0.02D + 3\ (Eqn\ 13.1a\ and\ 13.1b))$$

- Determine the diameter of the bolts from Eqn. 13.2c

$$T = \frac{\tau\pi i d^2 D_2}{8}$$

- Compressive stress in the bolt from Eqn 13.2d

$$T = \frac{\sigma_b i t d D_2}{2}$$

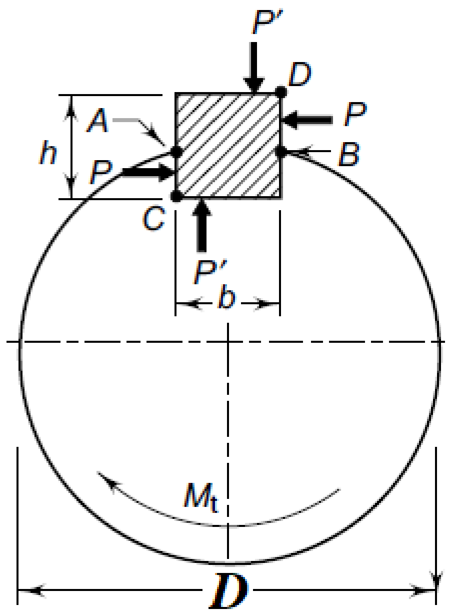
- Dimensions of keys (Page 61)

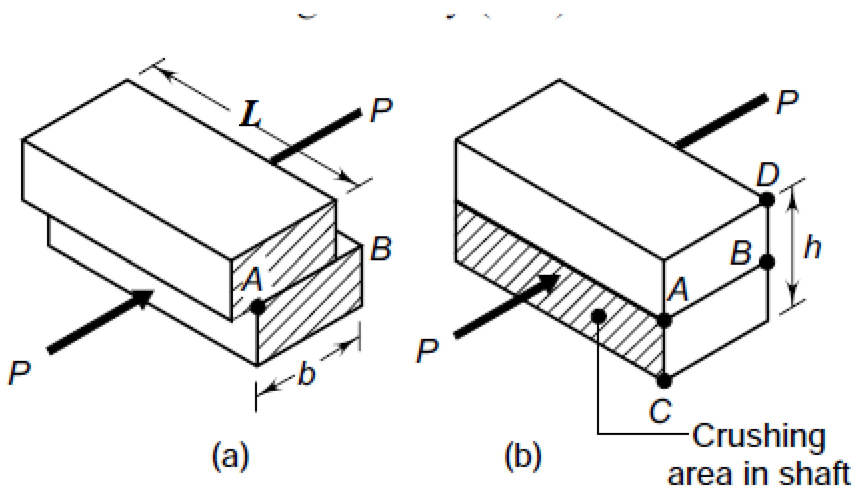
1. The width of the square key , $b = \frac{1}{4}d$ Page 61 Eq. 4.1
2. The width of the rectangular key, $b = \frac{1}{4}d$ Page 61 Eq. 4.2 a
3. The height of the rectangular key, h $= \frac{1}{6}d$ Page 61 Eq. 4.2 b
4. Refer table 4.1, data book page 69

- Check for shear and compressive stresses $\tau = \frac{2T}{DbL}$, $\sigma_c = \frac{4T}{DhL}$

Design Procedure: Rigid Flange Coupling

Forces acting on key





Problem 6.6

A rigid coupling is used to transmit 20 kW power at 720 rpm. There are four bolts and the pitch circle diameter of the bolts is 125 mm. The bolts are made of steel 45C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 3. Determine the diameter of the bolts.

Assume that the bolts are finger tight in reamed and ground holes.

Problem 6.7

It is required to design a rigid type of flange coupling to connect two shafts. The input shaft transmits 37.5 kW power at 180 rpm to the output shaft through the coupling. The service factor for the application is 1.5, i.e. the design torque is 1.5 times of the rated torque. Select suitable materials for various parts of the coupling and design the coupling and specify the dimensions of its components

Flexible Coupling

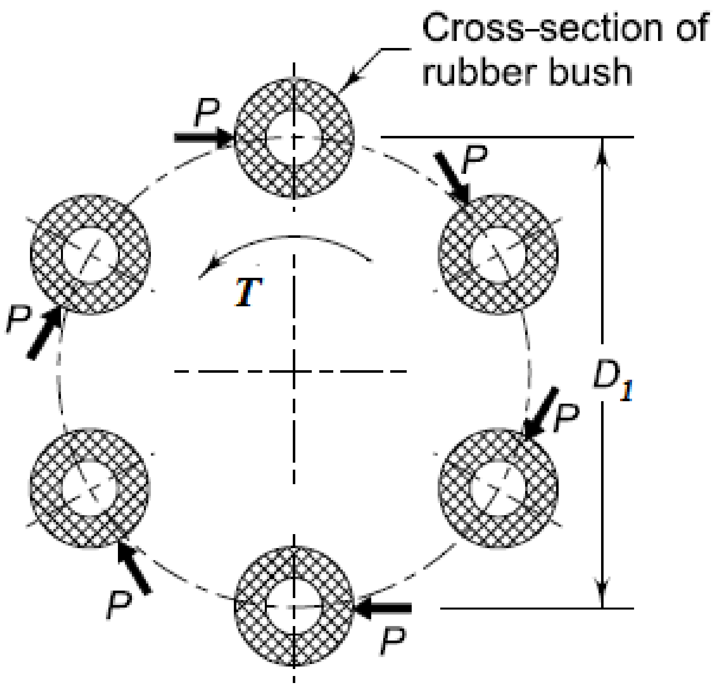
- Can tolerate 0.5 mm of lateral or axial misalignment and 1.50 of angular misalignment
- Absorbs vibrations
- Used for transmitting high torques

Flexible Coupling: Animation

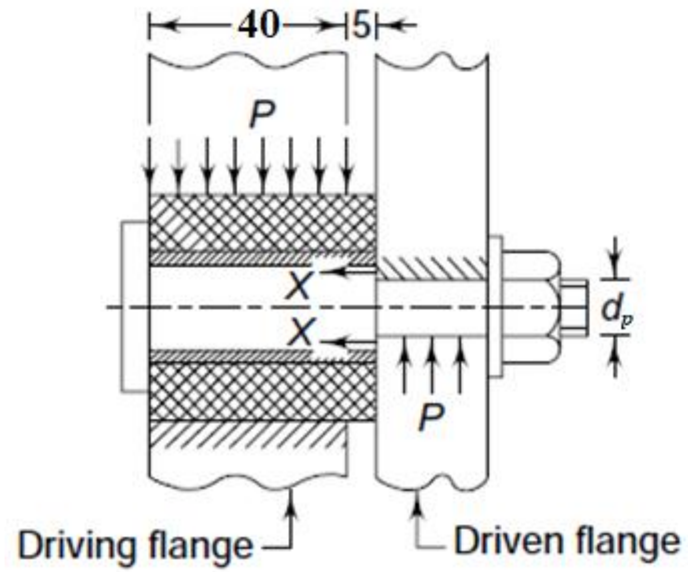
Problem 6.8

It is required to design a bushed pin type flexible coupling to connect the output shaft of an electric motor to the shaft of a centrifugal pump. The motor delivers 20 kW power at 720 rpm. The starting torque of the motor can be assumed to be 150% of the rated torque. Design the coupling and specify the dimensions of its components.

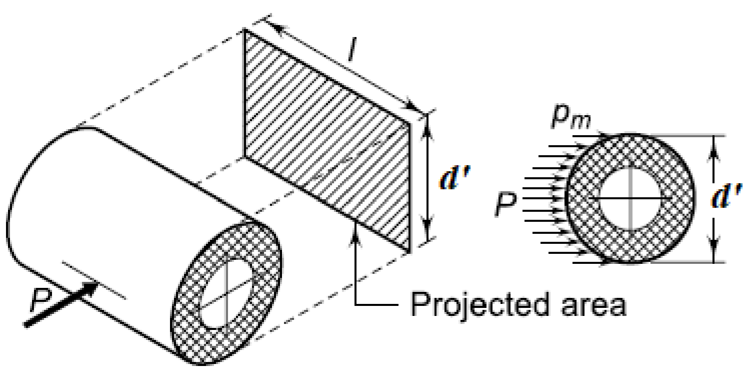
Problem 6.8



Cross-section of rubber bush
P
P
P
P
P
P
T
D_1



40
5
P
X
X
d_p
P
Driving flange
Driven flange



l
d'
p_m
P
d'
P
Projected area